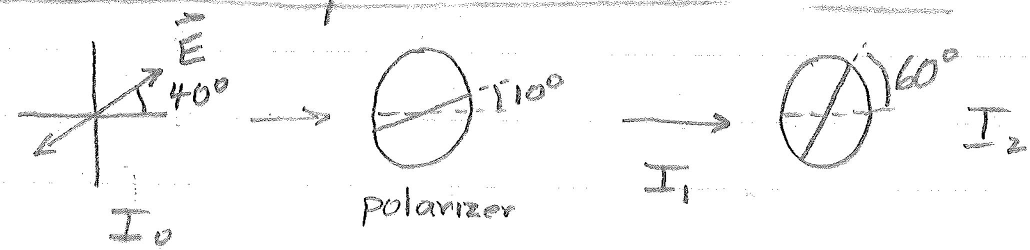
Physics 302 Photonics

HW 9 Chapt 8 - SOLUTIONS

(8.16) 

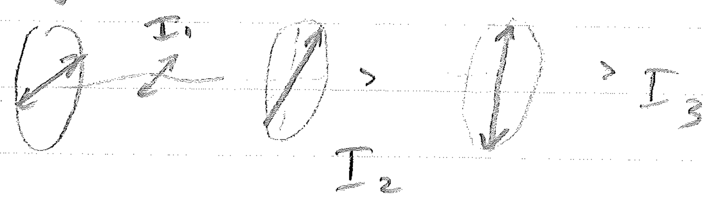
$\Rightarrow \quad I_1 = I_0 \cos^2(40° - 10°) = 0.75\, I_0$

I_1 is at $\theta_2 = (60° - 10°) = 50°$ wrt 2nd polarizer

$\Rightarrow \quad I_2 = I_1 \cos^2\theta_2 = (0.75 I_0)(\cos^2 50°)$

$\boxed{I_2 = 0.310\, I_0}$ ie <u>31% emerges</u>

(8.17) Question should be: "What percentage of light emerges from 2nd polarizer?"



$I_2 = I_1 \cos^2 45°$

$I_3 = I_2 \cos^2 45° = I_1 \cos^4 45°$

$\boxed{I_3 = 0.25\, I_1}$, ie <u>25% emerges</u>

(8.32) $V = \frac{V_P}{I_P + I_N}$

But $R_{\parallel} = \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)}$

$R_{\perp} = \frac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)}$

Since incident light is "natural" (unpolarized)

$I_{\parallel i} = I_{\perp i} = I_0/2$

(The reflected irradiance is): $I_{r\parallel} = R_{\parallel} I_{\parallel i} = R_{\parallel} I_0/2$

$I_{r\perp} = R_{\perp} I_{\perp i} = R_{\perp} I_0/2$, Also $I_{total} = I_{\parallel i} + I_{\perp i} = I_0$

So we need to calculate two "V"'s:

$V_{\parallel} = \frac{I_{r\parallel}}{I_{total}} = \frac{R_{\parallel} I_0/2}{I_0} = \frac{R_{\parallel}}{2}$, and $V_{\perp} = \frac{I_{r\perp}}{I_{total}} = \frac{R_{\perp} I_0/2}{I_0} = \frac{R_{\perp}}{2}$

To calculate $R_{\parallel}$ and $R_{\perp}$, we need θ_t from Snell's law:

$1 \sin\theta_i = n \sin\theta_t \Rightarrow \theta_t = \sin^{-1}\left(\frac{\sin\theta_i}{n}\right) = \sin^{-1}\left(\frac{\sin 40^\circ}{1.5}\right) = 25.37^\circ$

$\Rightarrow R_{\parallel} = \frac{\tan^2(40^\circ - 25.37^\circ)}{\tan^2(40^\circ + 25.37^\circ)} = \frac{0.06814}{4.756} = 0.0143$

$R_{\perp} = \frac{\sin^2(40^\circ - 25.37^\circ)}{\sin^2(40^\circ + 25.37^\circ)} = \frac{0.06379}{0.8263} = 0.0772$

$\Rightarrow$ $\boxed{V_{\parallel} = 7.16 \times 10^{-3}}$ $\boxed{V_{\perp} = 0.0386}$

(8.51) From eq. 8.44

$$V_{\frac{\lambda}{2}} = \frac{\lambda_0}{2n_o^3 r_{63}} = \frac{550\,nm}{2(1.58)^3(5.5\times10^{-12}\,\frac{m}{V})} = 12.8\,kV$$

(8.71) $\mathbb{E}_t = \mathbb{A}\,\mathbb{E}_i$ where $\mathbb{E}_i = \begin{pmatrix} E_{ix} \\ E_{iy} \end{pmatrix}$

a) Consider action of $\mathbb{A}_1 = \frac{1}{\sqrt{2}} e^{-i\pi/4} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$

on various initial polarization vectors.

$\mathbb{E}_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, ie $\mathcal{P}_{vertical}$. We can drop the normalization factors in $\mathbb{A}$ and $\mathbb{E}_i$

i) $\mathbb{E}_t = \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ i \end{pmatrix}$ ie $\mathcal{P}_{Vert} \xrightarrow{\mathbb{A}_1} \mathcal{L}_{state}$

ii) $\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} i \\ 1 \end{pmatrix} = \text{const}\begin{pmatrix} 1 \\ -i \end{pmatrix}$ ie $\mathcal{P}_{Hor.} \xrightarrow{\mathbb{A}_1} \mathcal{R}_{state}$

iii) $\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -i \end{pmatrix} = \begin{pmatrix} 1+1 \\ i-i \end{pmatrix} = \text{const}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ie $\mathcal{R}_{state} \xrightarrow{\mathbb{A}_1} \mathcal{P}_{vert}$

iv) $\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} 1-1 \\ i+i \end{pmatrix} = \text{const}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ie $\mathcal{L}_{state} \xrightarrow{\mathbb{A}_1} \mathcal{P}_{Hor.}$

This is some combination of wave plates, ie it is some combination of multiples of Jones matrices — I am not sure what combination but the characterization of the action of $\mathbb{A}_1$ is summarized ie i) Thru iv) above.

(8.71) b) Consider the action of $\mathbb{A}_2 = \frac{1}{\sqrt{2}} e^{i\pi/4} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$ on various initial polarization vectors. Again we can drop the normalization factors

i) $\mathbb{E}_t = \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -i \end{pmatrix}$ ie $\underline{P_{\text{vert}} \xrightarrow{\mathbb{A}_2} R_{\text{state}}}$

ii) $\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -i \\ 1 \end{pmatrix} = \text{cons}\begin{pmatrix} 1 \\ i \end{pmatrix}$ ie $\underline{P_{\text{Hor}} \xrightarrow{\mathbb{A}_2} L_{\text{state}}}$

iii) $\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -i \end{pmatrix} = \begin{pmatrix} 1-1 \\ -i-i \end{pmatrix} = \text{const}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ie $\underline{R_{\text{State}} \xrightarrow{\mathbb{A}_2} P_{\text{Hor}}}$

iv) $\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}\begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} 1+1 \\ -i+i \end{pmatrix} = \text{const}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ie $\underline{L_{\text{state}} \xrightarrow{\mathbb{A}_2} P_{\text{verT}}}$

Thus the characterization of $\mathbb{A}_2$ is summarized in i) thru iv) above.